Application of wavelet theory to the analysis of gravity data.

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Analysis of gravity data in the wavelet domain

Summary.

The fundamental equations of potential field theory have an interesting interpretation when regarded as a particular case of a multiscale wavelet transform. This allows the implementation of algorithms for image processing and inversion in a parameterisation that is geologically natural and meaningful.

Introduction.

Drawing upon recent results in the wavelet processing of images (e.g. Mallat and Zhong, 1992) and of potential fields (e.g. Hornby, Boschetti, and Horowitz, 1997), we have synthesized new methods of analyzing gravity fields, based upon a generalization of the concept of *edges*. Singularities in the mass density distribution of the sources are assumed to correspond one-to-one with apparent edges in the gravity field at different levels of upward continuation. Apparent edges in the field are defined to be local extrema of the horizontal gradient, which correspond nicely with the first stages of some traditional hand-drawn interpretation techniques. Upward continuation is used as the change of scale operation in defining the appropriate wavelet. The collection of apparent edges at all scales are termed a skeletonization of the field. Some properties of the skeletonization are (a) the variation of amplitude with position contains recoverable information about the type and location of the source density singularity, and (b) the field can, in most cases, be reconstructed from the skeletonization.

We use the Green's function of the gravity field as the mother wavelet in the multiscale wavelet analysis. This allows us to reinterpret traditional processing tools (such as upward and downward continuation) in a novel, elegant and rigorous manner. Algorithms for image processing and inversion may thus be based upon a parameterisation that is geologically meaningful (the lines commonly drawn by geologists over gravity images) rather than upon mathematically abstract and geologically unrealistic parameterizations, such as Fourier bases, rectangular voxels and so on. We believe this could also facilitate and encourage a closer collaboration and exchange of ideas between field geologists and mathematical geophysicists.

Method.

Choose a non-negative differentiable function $\theta(x)$ such that $\int_{\Re}\theta(x)\,dx = 1$; enforce the scaling property $\theta_s(x) = \frac{1}{s}\theta(\frac{x}{s})$; and define the function $\psi(x) = \frac{\partial\theta(x)}{\partial x}$ such that $\psi_s(x) = \frac{1}{s}\psi(\frac{x}{s})$. We now have $\theta(x)$ as a scaling function and $\psi(x)$ as a wavelet in a wavelet analysis.

The wavelet transform of a function f(x) is defined by

$$W[f](s,x) = (f * \psi_s)(x)$$

where $(*)$ denotes convolution. Then

$$W[f](s,x) = (f * \psi_s)(x) = f * (s\frac{\partial\theta_s}{\partial x})(x) = s\frac{\partial}{\partial x}(f * \theta_s)(x)$$

This shows that the wavelet transform $W[f](s,x)$ is, except for a scale factor, the first derivative of the signal smoothed at the scale s. The local extrema of $W[f](s,x)$ (in x) thus correspond to rapid variations in $(f * \theta_s)(x)$, and are collectively called **multiscale edges**.

Let us consider the magnitude of the gravitational vertical acceleration

$$f_{z_0}(x,y) = -g_z = \frac{\partial V}{\partial z} =$$

$$G\int_{\Re^2} dx'\,dy' \int_{-\infty}^{0} \frac{\rho(x',y',z')(z_0 - z')\,dz'}{((x-x')^2 + (y-y')^2 + (z_0 - z')^2)^{3/2}}$$

The Green's function for the gravitational vertical acceleration is:

$$\gamma(x,y,z) = \frac{z}{(x^2 + y^2 + z^2)^{3/2}}$$

Whose integral, divided by 2π is unity for all z > 0. Also, defining $\theta^{(sz)}(x,y) = \gamma(x,y,sz)/2\pi$, we have

$$\theta^{(sz)}(x,y) = \frac{1}{2\pi}\frac{sz}{(x^2 + y^2 + (sz)^2)^{3/2}} = \frac{1}{s^2}\theta^{(z)}(\frac{x}{s},\frac{y}{s})$$

The functions $^{x}\psi^{(z)} = \frac{\partial\theta^{(z)}}{\partial x}$ and $^{y}\psi^{(z)} = \frac{\partial\theta^{(z)}}{\partial y}$ share the property

$$^{i}\psi_s^{(z)}(x,y) = \frac{1}{s^2}\,{}^{i}\psi^{(z)}(\frac{x}{s},\frac{y}{s})$$

thus forming a set of self consistent dilation equations, and can be employed as wavelets.

Applications

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Here we briefly review some applications of the above theory to gravity surveys.

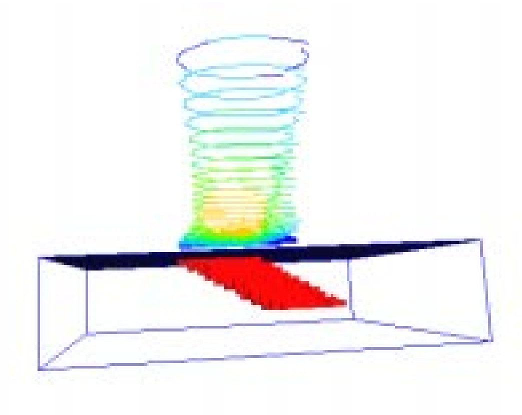

Figure 1. 3-D visualisation of the multiscale edges due to a dipping cylinder. Notice the effect of the dip on the bending of the edges at different height.

Behaviour of multiscale edges for simple geological shapes.

Figure 1 shows a 3-D visualisation of the multiscale edges corresponding to the gravity field due to a dipping cylinder of anomalous density. Mallat and Zhong (1992) show that the set of multiscale edges can be used to reconstruct the gravity image. This new representation allows an immediate understanding of some aspects of the shape of the causative body. The most obvious feature is the variation of the location of the edges at different levels as a function of the dip of the cylinder. Figure 2 shows a vertical section of the evolution of edges at different scales for faults of varying dip. The edges give a clear visual indicator of the direction of dip and also of the dip angle. In particular, vertically dipping density variations have vertical edges. These simple observations show the usefulness of this technique even for a first pass visual inspection.

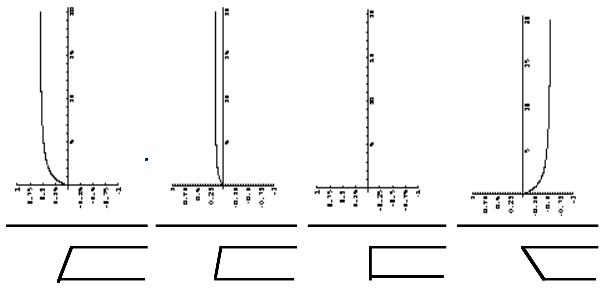

Figure 2. From the evolution of edges at different scales it is easy to determine the direction of dip and, to a lesser extent, the inclination of a fault.

Inversion and sensitivity to noise.

The evolution of edges at different scales contains information about the kinds of source singularities underlying a data set.. This property can be used in an inverse procedure to recover the shape and location of anomalous bodies from gravity fields. For example, we can invert the synthetic profiles shown in Figure 2 for the depth and dip of the causative faults. The results are reported in Tables 1 for an inversion after 20% of random noise was added to the synthetic data.

	Density	**Dip**	**Depth to Top**	**Depth to Bot.**
Synthetic	0.2	90	-1.0	-4.0
GA solution	0.23	90	-1.0	-3.6
Loc. Optim.	0.23	88.6	-0.66	-3.12

Table 1. Inversion of synthetic profiles, after adding 20% of random noise to the data. The density contrast, dip, depth to the top and to the bottom for a fault is given in the first row. The result of the inversion with a Genetic Algorithm is given in the second row. The third row gives the local optimisation of the GA solution. The fault is recovered with good accuracy.

Figure 3. Synthetic gravity image (a), edges at the finest scale (b) and image reconstructed from the multiscale edges only (c).

Image Processing.

The original image can be reconstructed from the multiscale edges (see Mallat and Zhong, 1992 for details). An example is given in Figure 3. Figure 3a shows a synthetic gravity image. Figure 3b displays the edges at the finest resolution. Figure 3c shows the reconstructed image. By manupulating a set of multiscale edges we can eliminate or locally enhance specific features in the image. This is a local process that does not affect the overall image, unlike analogous Fourier operations. Data compression and noise filtering are two other applications that could be derived from this property. We are implementing such algorithms, and hope to have examples from real images by the time of the meeting.

3-D visualisation and geological interpretation of skeletonizations.

Since the multiscale edges coincide with the main features in a map, they are a significant aid to the visual inspection of gravity maps. The technique is also applicable to magnetic fields, after a pseudogravity transformation is used to remove their dipole character. An example is given in Figure 4a, a magnetic data set collected by the Geological Survey of Victoria in west ern Victoria (Australia), and in Figure 4b, the edges at the finest scale after pseudogravity transformation. Notice how large areas in the magnetic image appear featureless, but the edges are able to detect highly detailed structure. A geologist familiar with the region has verified the accuracy of these edges.

Another example is given in Figure 5. It displays the skeletonization of the DNAG gravity field of North America. In our experience field geologists found this kind of visualisation particularly inspiring, especially for tectonic, large scale analysis. They have also provoked very productive exchanges of ideas with other geophysicists.

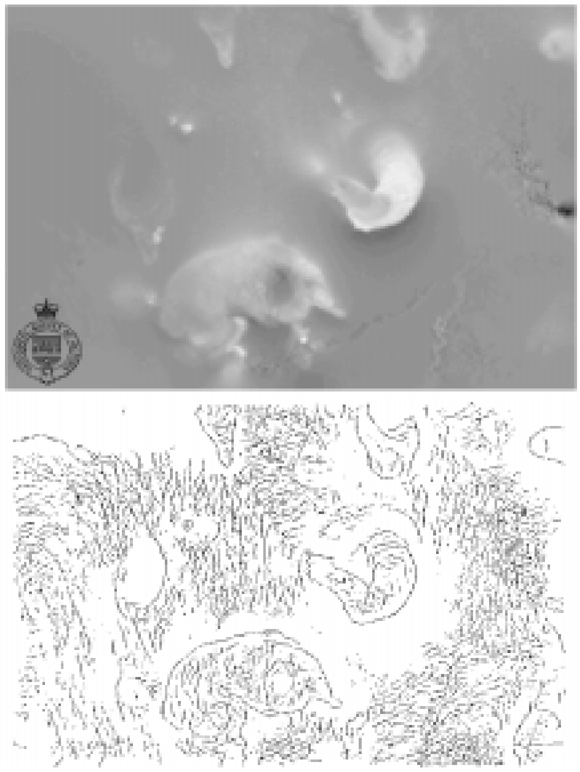

Figure 4. Aeromagnetic map collected by AGSO in west Victoria, Australia (top). Edges at the finest scale after pseudo gravity transformation (bottom). Notice the fine structure detected by the edges in areas that appear featureless to visual analysis in the original map.

We believe that the use of edges as a natural parameterisation of the problem is extremely useful in developing a common framework for interaction between geoscientists of different backgrounds.

Conclusions

We have presented a new set of tools for the analysis, processing and inversion of gravity data. We believe

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that their main advantage lies in the parameterisation that is natural to the problem and is also common to experts in the visual interpretation of potential field maps. We found that this gives a framework for a very productive communication and cooperation between geoscientists of different backgrounds.

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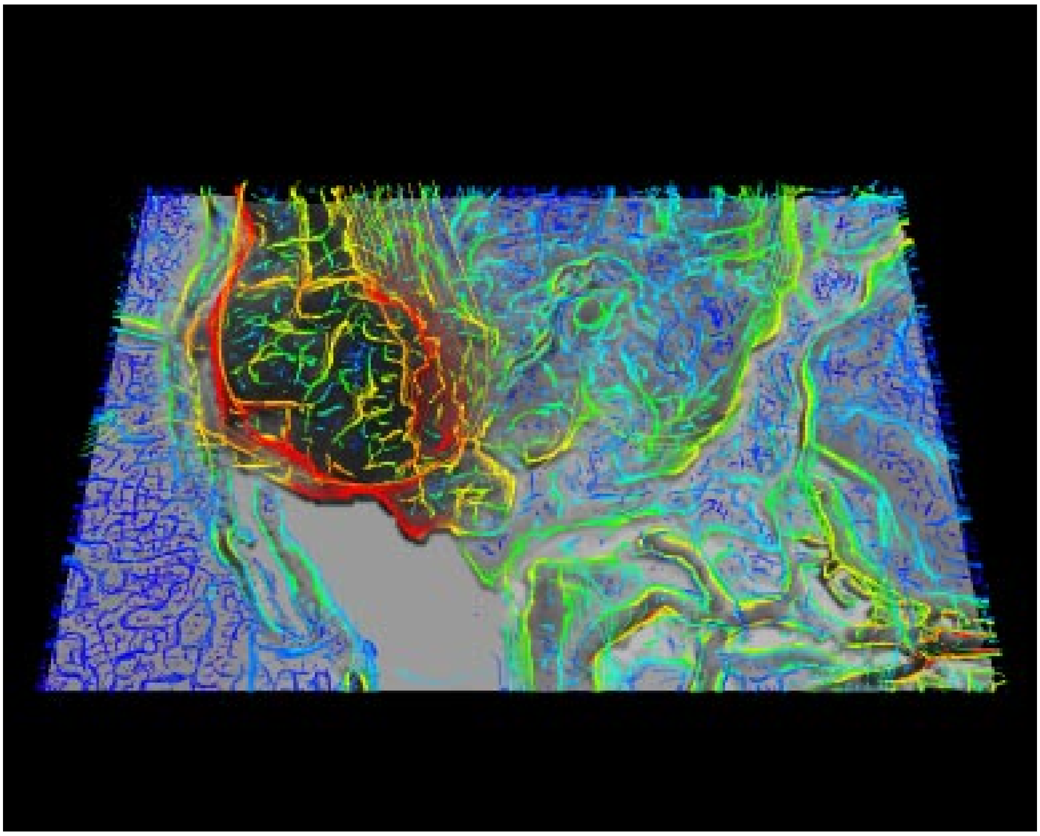

Figure 5. Skeleton map of the gravity field over USA. The edges at different scales are stacked one upon another. The visual inspection of the edges and their evolution at different scales gives indication of the major structures in the map and in some cases, of the geological setting of the causative sources.

References

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